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Evaluating the Steady-state Performance of the Synthetic Coefficient of Variation Chart

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ABSTRACT

The synthetic coefficient of variation (CV) chart is attractive to practitioners as it allows for a second point to fall outside the control limits before deciding whether the process is out-of-control. The existing synthetic CV chart is designed with a head-start feature, which shows an advantage under the zero-state assumption where shifts happen immediately after process monitoring has started. However, this assumption may not be valid as shifts may happen quite some time after process monitoring has started. This is called the steady-state condition. This paper evaluates the performance of the chart under the steady-state condition. It is shown that the steady-state out-of-control average run length (ARL_1) is substantially larger than the zero-state ARL_1, hence larger number of samples are needed to detect the out-of-control condition. From the comparison with other CV charts, the steady-state synthetic CV chart does not show better performance, especially for small sample sizes and shift sizes. Hence, the synthetic CV chart is not recommended to be adopted under the steady-state condition, and its

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good performance is only applicable under the zero-state assumption. The results of this paper enable practitioners to be aware that the performance of the synthetic CV chart may be inferior under actual application (when shifts do not happen at the beginning of process monitoring) compared to its zero-state performance.

Keywords: Coefficient of variation; control chart, exponentially weighted moving average, run rules, Shewhart, steady-state, synthetic chart, zero-state

INTRODUCTION

Control charts are useful tools to monitor a process for the presence of assignable cause(s), which results in an out-of-control condition. A lot of studies to improve the performance of control charts and to apply them in various scenarios are available in the literature, some of the recent ones are Khan et al. (2017), Marchant et al. (2018), You (2018), Mim et al. (2019) and Kinat et al. (2020). Similar with most conventional control charts, the charts are designed to monitor shifts in the process mean (μ) and/or standard deviation (σ), usually through $\bar{X}$ and/or R/S charts. However, control charting techniques are extended to various scientific areas, where μ and σ are always not constant and the process is nevertheless considered as in-control. This setting is usual in the context of engineering, healthcare, agriculture, education, and a variety of applications where the process output changes from time to time. For example, to maintain quality control checks on laboratory measurement on the amount of chemical in a patient's blood, the mean amount varies from patient to patient, making it unsuitable to monitor μ.

Conventional μ and/or σ charts cannot be applied to monitor the stability and variability for such processes, since a change in μ and/or σ does not show an out-of-control (OOC) process. For processes with an inconsistent μ and/or σ, it is a better alternative to monitor the CV= $\left(\gamma = \frac{\sigma}{\mu}\right)$ if the ratio of σ to μ is consistent, even though μ and/or σ varies. Yeong et al. (2017) have reviewed several fields where the CV is important. For example, σ is often found to be proportional with μ for certain quality characteristics related to the physical properties of metal alloys or composite materials, for example tool cutting life and several properties of sintered materials (Castagliola et al., 2011). In investments, the CV can measure the volatility compared to the expected return (Curto & Pinto, 2009). For example, Pang et al. (2008) measured the stability of dividend yields of the Hang Seng index through γ.

Kang et al. (2007) was the pioneer who first proposed a Shewhart type chart to monitor the CV. Subsequently, several new CV charts are proposed to improve the performance of CV-type charts, for example the Exponentially Weighted Moving Average (EWMA) CV chart by Castagliola et al. (2011) and the synthetic CV chart by Calzada and Scariano (2013). Recently, Zhang et al. (2018) proposed an improved EWMA CV chart by truncating negative normalized observations to zero and Mahmood and Abbasi (2021) improved the performance of the Shewhart CV chart under neoteric ranked set sampling.

The synthetic CV chart by Calzada and Scariano (2013) modifies the synthetic chart by Wu and Spedding (2000), who proposed the synthetic chart to monitor the process mean, so that it can monitor the CV. Subsequently, Yeong et al. (2018) proposed the economic and economic-statistical designs of the synthetic CV chart. The synthetic CV chart shows smaller ARL_1 than the Shewhart CV chart in detecting all shifts but shows larger ARL_1 than the EWMA CV chart. The ARL is a commonly used measure of performance for control charts (Montgomery, 2019). There are two types of ARL, i.e., the IC ARL (ARL_0) and OOC ARL (ARL_1). The ARL_0 measures the average number of samples taken until a false alarm occurs, while the ARL_1 measures the average number of samples taken until an OOC condition is detected. A large ARL_0 and a small ARL_1 is preferred. By fixing the ARL_0 as a specific value, a chart shows better performance if it has a small ARL_1. This is because a chart with a smaller ARL_1 can detect shifts faster.

For a synthetic CV chart, when a sample CV $(\hat{\gamma})$ falls outside the upper and lower control limits (UCL and LCL, respectively), an OOC signal is not immediately produced. Instead, these samples are only known as non-conforming samples, while conforming samples are samples within the limits. The chart will signal depending on the conforming run length (CRL), which counts the number of conforming samples between successive non-conforming samples. When $\text{CRL} \leq L$, the process is OOC, conversely it is in-control (IC). Note that L is a threshold set by the practitioner, which determines how close two successive non-conforming samples must be to each other (measured in terms of the number of conforming samples between two successive non-conforming samples), so that an OOC signal will be produced. Since the synthetic CV chart gives an OOC signal when $\text{CRL} \leq L$, a smaller value of L indicates that the non-conforming samples must be quite close to each other to produce an OOC signal, while for larger values of L, an OOC signal will still be produced even though the non-conforming samples are quite far from each other. A larger L is usually selected when practitioners are interested to detect a small shift in the process, conversely a smaller L is selected for large shifts. In practice, the value of L is determined to minimize the ARL_1, subject to constraints in the ARL_0.

Recently, Rakitzis et al. (2019) gave an overview of recent studies on synthetic-type charts. A thorough review is provided for different types of synthetic chart monitoring the mean, variance and the joint monitoring of the mean and variance. Rakitzis et al. (2019) has stated that the synthetic chart is designed with a head-start feature which leads to misleading conclusion on the actual performance of the synthetic chart. Similarly, the synthetic CV chart by Calzada and Scariano (2013) is also designed based on an enormous head-start. This head-start feature assumes that a non-conforming sample is present at the starting time (t = 0). As a result, the first CRL will simply be the number of samples until the first non-conforming sample is observed. If the first CRL is less than or equals to L, then this head-start feature will provide an advantage to the synthetic CV chart, since fewer samples are required to give an OOC signal as the chart does not have to wait until the second sample to fall outside the control limits before deciding whether the process is IC or OOC. If the process starts in an OOC condition (commonly referred to as the zero-state condition), it is likely that the first non-conforming sample will be encountered not long after process monitoring has started, hence it is likely that the first CRL will be less than or equal to L. Under such conditions, the head-start feature will result in an improvement in the performance of the synthetic CV chart.

However, when the process shifts only happen after the process has been operating for quite some time (commonly referred to as the steady-state condition), it is unlikely for a non-conforming sample to be encountered not long after the starting time. As a result, it is likely that the first CRL will be more than L, and the head-start feature will not result in an advantage anymore. In this paper, the performance of the synthetic CV chart when the head-start advantage has faded away will be studied. This is important so that practitioners can evaluate the performance of the synthetic CV chart without the head-start advantage, which is likely to occur in a steady-state condition.

Davis and Woodall (2002) were the pioneer who propose the steady-state synthetic $\bar{X}$ chart. Recently, Knoth (2016) conducted a thorough study on the synthetic $\bar{X}$ chart by considering two steady-state assumptions, i.e., conditional, and cyclical steady-states. Conditional steady-state assumes that there are no false alarms before the shift, while the cyclical steady-state assumes that a series of false alarms may happen before the shift. Formulae for the conditional and cyclical ARLs are derived. There is a significant difference between the zero-state and steady-state performance of the synthetic $\bar{X}$ chart. For small and moderate shifts, the zero-state synthetic $\bar{X}$ chart performs better than the run rules chart, while for large shifts, it performs better than the EWMA chart. However, the synthetic $\bar{X}$ chart shows the worst steady-state performance compared to the steady-state EWMA and run rules charts (Knoth, 2016).

Teoh et al. (2016) compared the cyclical steady-state performance of the synthetic and EWMA CV charts. However, the methodology to evaluate the cyclical steady-state performance is not given. Furthermore, the conditional steady-state performance is not studied in Teoh et al. (2016). This paper provides the formulae for the conditional and cyclical steady-states, so that practitioners can easily evaluate the steady-state performance of the synthetic CV chart. The methodology to obtain the optimal chart parameters based on the steady-state performance is also provided. Furthermore, this paper also evaluates the impact of different assumptions on initial states towards the ARL, which is not studied in Teoh et al. (2016). The initial state refers to the state prior to the shift in the process. This paper considers two designs. Both designs evaluate the steady-state performance. In the first design, chart parameters that optimizes the zero-state ARL are adopted, while the second design optimizes the steady-state ARL. These two designs are, then, compared to evaluate whether the second design results in a significant improvement compared to the first design.

MATERIALS AND METHODS

This section starts with a review of the operations of the synthetic CV chart. Subsequently, the zero and steady-state ARL of the synthetic CV chart is discussed.

The synthetic CV chart classifies a sample as non-conforming if the sample CV is either above the upper control limit (UCL) or below the lower control limit (LCL), i.e., $\hat{\gamma} < \text{LCL}$ or $\hat{\gamma} < \text{LCL}$. By letting "0" denote a conforming sample and "1" denote a non-conforming sample, a series of samples can be illustrated as a sequence of zeros and ones. For example, 10010 shows a series of five samples, where the non-conforming samples are the first and fourth samples. The digits to the right show whether the most recent sample is conforming/non-conforming, while digits to the left shows whether the earlier samples are conforming or non-conforming.

When two successive "1"s are encountered, the number of samples between the successive "1"s are defined as the CRL, where the CRL includes the ending non-conforming sample. For example, the CRL is 4 for string 10001. When $\text{CRL} \leq L$, the process is OOC, conversely it is IC. For example, the string 10001 is IC if L is set as 3, while it is OOC if L is set as 4, or any value more than 4.

When two successive non-conforming samples are encountered, the synthetic chart determines whether the process is IC or OOC by counting the number of conforming samples between successive non-conforming samples. However, when the first non-conforming sample is encountered, the CRL will simply be the number of samples until this non-conforming sample is encountered. This is called the head-start feature. For example, CRLs for the initial strings 1, 01, 001 and 0001 are 1, 2, 3 and 4, respectively. If L is set as 3, the first three strings will result in an OOC signal, while the fourth string results in an IC signal.

The head-start feature will result in a faster detection under a zero-state assumption. However, under the steady-state condition (where the process becomes OOC after the process has been operating for some time), the head-start advantage would have faded away. Hence, it is important for practitioners to evaluate the steady-state performance of the synthetic CV chart without the head-start advantage.

Zero-state ARL

Castagliola et al. (2011) has shown that the cumulative distribution function (cdf) of $\hat{\gamma}$ in Equation 1.

$$F_{\hat{\gamma}}\left(x \middle| n,\gamma\right) = 1 - F_t\left(\frac{\sqrt{n}}{x} \middle| n-1, \frac{\sqrt{n}}{\gamma}\right) \quad [1]$$

Where $F_t\left(\cdot \middle| n-1, \frac{\sqrt{n}}{\gamma}\right)$ is the non-central t-distribution with (n–1) degrees of freedom and non-centrality parameter $\frac{\sqrt{n}}{\gamma}$, with n being the sample size and γ the CV.

Inverting the cdf in Equation 1 gives Equation 2,

$$F_{\hat{\gamma}}^{-1}(\alpha|n,\gamma) = \frac{\sqrt{n}}{F_t^{-1}\left(1-\alpha \middle| n-1, \frac{\sqrt{n}}{\gamma}\right)} \quad [2]$$

with $F_t^{-1}\left(\cdot \middle| n-1, \frac{\sqrt{n}}{\gamma}\right)$ being the inverse cdf of the non-central t distribution. From Equation 1 and 2, if $F_{\hat{\gamma}}^{-1}\left(\alpha \middle| n,\gamma\right) = x$, then $F_{\hat{\gamma}}\left(x \middle| n,\gamma\right) = \alpha$. In other words, $F_{\hat{\gamma}}^{-1}\left(\alpha \middle| n,\gamma\right)$ evaluates the value of x such that $P\left(\hat{\gamma} \le x\right) = \alpha$. In control chart design, α is usually related to the false alarm probability, so that the control limits can be determined to obtain a false alarm probability set by the practitioner.

The LCL and UCL can be obtained as Equation 3,

$$\text{LCL} = \frac{\sqrt{n}}{F_t^{-1}\left(1-\frac{p}{2} \middle| n-1, \frac{\sqrt{n}}{\gamma_0}\right)} \quad [3]$$

and Equation 4.

$$\text{UCL} = \frac{\sqrt{n}}{F_t^{-1}\left(\frac{p}{2} \middle| n-1, \frac{\sqrt{n}}{\gamma_0}\right)} \quad [4]$$

Where γ_0 is the IC CV and p is the probability the sample CV falls outside the LCL and UCL when the process is IC. Note that p is usually determined to fix the IC run length.

Let A and B be the probabilities a sample is conforming and non-conforming, respectively. A is obtained as Equation 5,

$$A = P\left(\text{LCL} < \hat{\gamma} < \text{UCL}\right)$$
$$= F_{\hat{\gamma}}(UCL|n,\gamma) - F_{\hat{\gamma}}(LCL|n,\gamma) \quad [5]$$

with $F_{\hat{\gamma}}(.)$ defined in Equation 1 and $B = 1 - A$. By merging Equation 1 and 5, A can be computed as Equation 6.

$$A = F_t\left(\frac{\sqrt{n}}{\text{LCL}} \middle| n-1, \frac{\sqrt{n}}{\gamma}\right) - F_t\left(\frac{\sqrt{n}}{\text{UCL}} \middle| n-1, \frac{\sqrt{n}}{\gamma}\right). \quad [6]$$

To construct the Markov chain, we define the states 0, 1, …, L-1 as the number of "0"s after the most recent 1, and state L as the state with at least L "0"s after the most recent 1. For example, for L = 3, the states 0, 1, 2 and 3 represent the initial strings 001, 010, 100 and 000, respectively. The transition probability matrix is constructed as Equation 7,

$$\mathbf{P} = \begin{array}{c} 0 \\ 1 \\ 2 \\ : \\ L-2 \\ L-1 \\ L \\ \text{OOC} \end{array} \begin{pmatrix} 0 & A & 0 & 0 & ... & 0 & 0 & B \\ 0 & 0 & A & 0 & ... & 0 & 0 & B \\ 0 & 0 & 0 & A & ... & 0 & 0 & B \\ : & : & : & : & : & : & : & : \\ 0 & 0 & 0 & 0 & ... & A & 0 & B \\ 0 & 0 & 0 & 0 & ... & 0 & A & B \\ B & 0 & 0 & 0 & ... & 0 & A & 0 \\ 0 & 0 & 0 & 0 & ... & 0 & 0 & 1 \end{pmatrix} \quad [7]$$

where OOC is the absorbing OOC state. Note that the states of the Markov chain in Equation 7 are defined differently from most synthetic papers. By referring to the state definitions in Davis and Woodall (2002), states 0, 1, …, L-1 in this paper is equivalent to states 1, 2, …, L in Davis and Woodall (2002), while state L in this paper is equivalent to state 0 in Davis and Woodall (2002). $\mathbf{P}$ in Equation 7 can be converted to the same form as that in Davis and Woodall (2002) by shifting the row and column for state L to the first row and column, respectively.

The rationale behind adopting slightly different state definitions is to facilitate a more intuitive analysis of the performance for the synthetic CV chart under different initial states. By referring to the state definitions in the preceding paragraph, as the states become larger, the non-conforming sample moves towards the left (except for State L where all the samples are conforming). This shows that the smaller states have a more recent occurrence of the non-conforming sample. By studying the performance of the chart as the states increases, practitioners will be able to easily observe the impact of the non-conforming sample's position on the performance of the chart.

By removing the last row and column of $\mathbf{P}$, a transient $(L+1)\times(L+1)$ sub-matrix $\mathbf{Q}$ is obtained. The ARL is computed as Equation 8.

$$ARL = \mathbf{q}^T (\mathbf{I}-\mathbf{Q})^{-1} \mathbf{1} \quad [8]$$

Where $\mathbf{q}$ is the $(L+1)\times 1$ vector of initial probabilities for the transient states, $\mathbf{I}$ is an $(L+1)\times(L+1)$ identity matrix and $\mathbf{1}$ is an $(L+1)\times 1$ vector of ones. Under the zero-state assumption, $\mathbf{q} = (1, 0, \ldots, 0)^T$. In addition, zero-state ARLs for all possible starting states can also be obtained by solving $(\mathbf{I}-\mathbf{Q})\boldsymbol{l} = \mathbf{1}$, where $\boldsymbol{l} = (l_0, \quad l_1, \quad \cdots \quad l_L)^T$ are the ARLs for the different starting states. By letting $r = B(1-A^L)$, Knoth (2016) has defined Equation 9.

$$\boldsymbol{l}^T = \begin{pmatrix} \overset{0}{\frac{1}{r}} & \overset{1}{\frac{1+A^L(A^{-1}-1)}{r}} & \overset{...}{...} & \overset{L-1}{\frac{1+A^L(A^{-(L-1)}-1)}{r}} & \overset{L}{\frac{1}{r}+\frac{1}{B}} \end{pmatrix}, \quad [9]$$

Where the ARL_0 is computed by letting $\gamma = \gamma_0$ in Equation 6 to obtain the A and B in Equation 9, while the ARL_1 is computed by letting $\gamma = \tau\gamma_0$, with τ being the shift size the practitioner is interested to detect. Note that although notionally different and with different state definitions as mentioned in the preceding paragraph, $\boldsymbol{l}^T$ in Equation 9 is the same as the ARL vector in Shongwe and Graham (2017; 2019). However, note that Shongwe and Graham (2017; 2019) conducted a study for the synthetic $\overline{X}$ chart, while this paper studies the synthetic CV chart.

Steady-state ARL

The distribution for different initial states before the change point (θ) is studied in this section. The distribution is based on conditional and cyclical assumptions, and it will be utilized in formulating the steady-state ARLs.

To obtain the conditional stationary density, let $\boldsymbol{\psi}^T = (\pi_0 \quad \pi_1 \quad \cdots \quad \pi_{L-1} \quad \pi_L)$ be the quasi-stationary density conditioned on no false alarm before θ, where π_i is the state i stationary probabilities, $i = 0, 1, \ldots, L$. From Markov chain theory, $\boldsymbol{\psi}^T$ can be obtained by solving the equations $\mathbf{Q}^T\boldsymbol{\psi} = \varphi\boldsymbol{\psi}$ and $\boldsymbol{\psi}^T\mathbf{1} = 1$, where $\boldsymbol{\psi}^T$ is obtained as Equation 10.

$$\begin{array}{cccccc} & 0 & 1 & \ldots & L-1 & L \\ \boldsymbol{\psi}^T = & \Bigg(s & \dfrac{A}{\varphi}s & \ldots & \left(\dfrac{A}{\varphi}\right)^{L-1} s & \dfrac{\varphi}{B}s \Bigg), \end{array} \quad [10]$$

Where $s = 1 - \dfrac{A}{\varphi}$. φ is obtained by solving $\boldsymbol{\psi}^T\mathbf{1} = 1$ numerically with the starting value $\varphi_0 = 1$.

To obtain the cyclical stationary density, it needs to be considered that the process restarts at the zero-state level after a false alarm. Hence, the matrix $\mathbf{Q}$ is modified by adding the contents of the last column of $\mathbf{Q}$ into the first column of $\mathbf{Q}$. We refer to this modified matrix as $\mathbf{Q}_*$. By adopting a similar approach as that for the conditional probability distribution $\boldsymbol{\psi}$, the cyclical stationary density is obtained as Equation 11,

$$\begin{array}{cccccc} & 0 & 1 & \ldots & L-1 & L \\ \boldsymbol{\psi}_* = & \big(B & AB & \ldots & A^{L-1}B & A^L \big), \end{array} \quad [11]$$

which is the cyclical stationary distribution for the synthetic chart. The cyclical stationary distribution for the synthetic chart was first discussed and formulated by Machado and Costa (2014). Note that Machado and Costa (2014) investigated the performance for the synthetic $\overline{X}$ chart, while this paper studies the performance of the synthetic CV chart.

From the conditional and cyclical distributions in Equation 10 and 11, the conditional and cyclical steady-state ARLs, denoted as ARL_{cond} and ARL_{cyc} respectively, is obtained as Equation 12,

$$= \left(\frac{\varphi_0}{B_0} + \frac{1-\left(\frac{A_0}{\varphi_0}\right)^L}{1-\frac{A_0}{\varphi_0}} \right) \frac{s_0}{B_\delta} + \left(\frac{\varphi_0}{B_0} + A_\delta^L \frac{1-\left(\frac{A_0}{A_\delta \varphi_0}\right)^L}{1-\frac{A_0}{A_\delta \varphi_0}} \right) \frac{s_0}{r_\delta}, \qquad [12]$$

and Equation 13.

$$\mathrm{ARL}_{\mathrm{cyc}} = \boldsymbol{\o}_{*,0}^{T} \boldsymbol{l}_\delta$$

$$= \frac{1 + A_0 B_\delta \frac{A_0^L - A_\delta^L}{A_0 - A_\delta}}{r_\delta}$$

$$= \frac{1}{B_\delta} + \left(1 - \frac{B_\delta A_0 \left(1 - A_0^L\right) - B_0 A_\delta \left(1 - A_\delta^L\right)}{A_0 - A_\delta} \right) \frac{1}{r_\delta} \qquad [13]$$

Where the subscripts 0 and δ denote the IC and OOC versions. The longer term in Equation 13 is like that of Wu et al. (2010). Note that Wu et al. (2010) studied the performance for the synthetic $\bar{X}$ chart, while this paper studies the performance of the synthetic CV chart.

RESULTS AND DISCUSSION

This section starts with a study on the performance of the synthetic CV chart under different initial states, and the probability for different initial states under the conditional and cyclical steady-state assumptions. The initial states refer to the state before the process shift. Next, the steady-state performance of the synthetic CV chart is studied based on two designs. Finally, the zero and steady-state performance of the synthetic CV chart is compared with the EWMA, run rules and Shewhart CV charts.

Synthetic CV Chart Under Different Initial States

In this section, the impact of different initial states on the performance of the synthetic CV chart is studied. Unlike the zero-state ARL which assumes that the initial state is zero, the actual initial state can belong to any of the states 0, 1, …, L. This section investigates the effects on the ARL_1 when the optimal chart parameters that minimize the zero-state ARL_1 is adopted on processes with a non-zero initial state. The probability for different initial states is also obtained. The effects on the ARL_1 and the initial state probabilities are evaluated based on the conditional and cyclical assumptions.

The optimal chart parameters and ARL_1 for $n = 5$ and $\tau \in \{1.10, 1.25, 1.50, 2.00\}$ from Calzada and Scariano (2013) is shown in Table 1. Note that the optimal chart parameters from Calzada and Scariano (2013) is based on the zero-state ARL_1.

The ARL_1 shown in Table 1 can only be obtained under a zero-state condition or if the initial state is 0. However, under the steady-state assumption, the initial state may not be zero, and the ARL_1 shown in Table 1 may not be obtained. For example, the zero-state ARL_1 for $i = 0,1,...,L$ is 115.39. However, the ARL_1 might not be 115.39 if the initial state is not 0. Hence, this section will look at the actual ARL_1 for different initial states. For an initial state *i*, $i = 0,1,...,L$, this is achieved by letting the $(i+1)^{th}$ element of the vector $\mathbf{q}$ be 1, while letting the other elements of $\mathbf{q}$ be zero. Table 2 shows the conditional and cyclical probabilities, as well as the ARL_1 for different initial states, when the optimal chart parameters in Table 1 is adopted, where similar with Table 1, n = 5 and $\gamma_0 = 0.05$. Due to space constraints and the consistency in the trends, only results for states 0,1, L-1 and L are shown in Table 2.

The same optimal chart parameters in Table 1 are adopted in Table 2. This is to study the effects of adopting these chart parameters for cases where the initial state is not zero. For example, for $n = 5, \gamma_0 = 0.05$ and $\tau = 1.10$, this paper investigates the impact towards ARL_1 when the optimal chart parameters $(L = 73, LCL = 0.01031, UCL = 0.09943)$, obtained to optimize the ARL_1 based on a zero initial state, is implemented on processes with a non-zero initial state, i.e. any initial state from states 1 to 73. This will enable us to study the actual performance of the chart when the initial state is not zero, as shown in Table 2.

Table 1

Optimal chart parameters and ARL_1 of the zero-state synthetic CV chart for n = 5, $\gamma_0 = 0.05$ and $\tau \in \{1.10, 1.25, 1.50, 2.00\}$

τ	L	LCL	UCL	ARL_1
1.10	73	0.01031	0.09943	**115.39**
1.25	30	0.01142	0.09651	**24.02**
1.50	12	0.01277	0.09326	**5.76**
2.00	5	0.01426	0.08993	**1.97**

By moving vertically down the second last column of Table 2, the ARL_1 increases gradually as the initial state becomes larger. The last column of Table 2 shows the percentage increase from the zero-state ARL_1. The differences between the ARL_1 for an initial state of zero and an initial state of L can be quite large, especially for small shift sizes of $\tau = 1.10$. For example, when $\tau = 1.10$, the ARL_1 increases by 63.39% from 115.39 when the initial state is zero to 188.53 when the initial state is 73. A similar trend is also shown for other values of τ. This shows that the actual performance will differ from that shown by the zero-state ARL_1 when the state before the change point is not zero.

Table 2
The conditional probability, cyclical probability and ARL_1 for different initial states based on the optimal chart parameters for the zero-state synthetic CV chart

τ	L	LCL	UCL	Initial state	Conditional Probability	Cyclical Probability	ARL_1	Percentage Increase from the Zero-state ARL_1
1.10	73	0.01031	0.09943	0	0.00482	0.00685	115.39	**0%**
				1	0.00480	0.00680	115.97	**0.50%**
				⋮	⋮	⋮	⋮	⋮
				72	0.00341	0.00418	186.96	**62.02%**
				73	0.70264	0.60547	188.53	**63.39%**
1.25	30	0.01142	0.09651	0	0.00802	0.01019	24.02	**0%**
				1	0.00796	0.01009	24.28	**1.08%**
				⋮	⋮	⋮	⋮	⋮
				29	0.00635	0.00757	41.95	**74.65%**
				30	0.78532	0.73536	43.20	**79.85%**
1.50	12	0.01277	0.09326	0	0.01335	0.01565	5.76	**0%**
				1	0.01317	0.01540	5.87	**1.91%**
				⋮	⋮	⋮	⋮	⋮
				11	0.01151	0.01316	9.96	**72.92%**
				12	0.85107	0.82757	11.05	**91.84%**
2.00	5	0.01426	0.08993	0	0.02142	0.02381	1.97	**0%**
				1	0.02096	0.02324	2.03	**3.05%**
				⋮	⋮	⋮	⋮	⋮
				4	0.01964	0.02162	2.87	**45.69%**
				5	0.89740	0.88649	3.89	**97.46%**

Steady-state Performance

This section evaluates the conditional and cyclical steady-state ARL_1s. The first design adopts chart parameters that are optimal for the zero-state ARL_1, while the second design adopts chart parameters that are optimal for the steady-state ARL_1.

First Design. The optimal chart parameters based on the methodology by Calzada and Scariano (2013), which optimizes the zero-state ARL_1, is adopted in this design. The value of p in Equation 3 and 4 is obtained so that the zero-state ARL_0 = 370.4. Table 3 shows the zero-state, conditional and cyclical steady-state ARL_1s based on these optimal chart parameters for $\gamma_0 \in \{0.05, 0.10, 0.20\}$, $n \in \{5, 10, 15\}$ and $\tau \in \{1.10, 1.25, 1.50, 2.00\}$. In parenthesis beside the steady-state ARL_1s are the percentage increase from the zero-state ARL_1. The first three columns of Table 3 show the optimal (L, LCL, UCL) which minimizes the zero-state ARL_1 in Equation 8, subject to the constraint ARL_0 = 370.4.

Table 3

The conditional and cyclical steady state ARL'_1 when chart parameters which optimizes the zero-state ARL_1 are adopted

τ	L	LCL	UCL	Zero-state ARL_1	Conditional Steady-state ARL'_1	**Cyclical Steady-state ARL'_1**
			$\gamma_0 = 0.05$			
			***n* = 5**			
1.10	73	0.01031	0.09943	115.39	175.10 (51.75%)	**170.37 (47.65%)**
1.25	30	0.01142	0.09651	24.02	40.47 (68.48%)	**39.81 (65.74%)**
1.50	12	0.01277	0.09326	5.76	10.47 (81.77%)	**10.37 (80.03%)**
2.00	5	0.01426	0.08993	1.97	3.73 (89.34%)	**3.71 (88.32%)**
			***n* = 10**			
1.10	57	0.02118	0.08237	78.87	122.40 (55.19%)	**119.56 (51.59%)**
1.25	17	0.02277	0.07975	11.48	20.03 (74.48%)	**19.82 (72.65%)**
1.50	6	0.02435	0.07727	2.71	5.05 (86.35%)	**5.03 (85.61%)**
2.00	3	0.02550	0.07552	1.22	2.35 (92.62%)	**2.34 (91.80%)**
			***n* = 15**			
1.10	46	0.02651	0.07554	58.48	92.21 (57.68%)	**90.36 (54.51%)**
1.25	12	0.02814	0.07304	7.18	12.50 (74.09%)	**12.39 (72.56%)**
1.50	4	0.02965	0.07109	1.86	3.50 (88.17%)	**3.49 (87.63%)**
2.00	2	0.03070	0.06968	1.07	2.06 (92.52%)	**2.05 (91.59%)**
			$\gamma_0 = 0.10$			
			***n* = 5**			
1.10	73	0.02057	0.20079	116.16	176.19 (51.68%)	**171.44 (47.59%)**
1.25	31	0.02271	0.19499	24.34	41.06 (68.69%)	**40.37 (65.86%)**
1.50	12	0.02549	0.18805	5.85	10.62 (81.54%)	**10.53 (80.00%)**
2.00	5	0.02846	0.18120	2.00	3.78 (89.00%)	**3.76 (88.00%)**

Table 3 *(Continued)*

τ	L	LCL	UCL	Zero-state ARL_1	Conditional Steady-state ARL'_1	**Cyclical Steady-state ARL'_1**
			$\gamma_0 = 0.05$			
			***n* = 10**			
1.10	59	0.04217	0.16590	79.77	124.06 (55.52%)	**121.08 (51.79%)**
1.25	17	0.04544	0.16038	11.71	20.37 (73.95%)	**20.15 (72.08%)**
1.50	6	0.04859	0.15533	2.76	5.15 (86.59%)	**5.12 (85.51%)**
2.00	3	0.05090	0.15175	1.24	2.37 (91.13%)	**2.37 (91.13%)**
			***n* = 15**			
1.10	46	0.05291	0.15180	59.32	93.43 (57.50%)	**91.57 (54.37%)**
1.25	12	0.05616	0.14702	7.33	13.04 (77.90%)	**12.94 (76.53%)**
1.50	4	0.05919	0.14273	1.89	3.56 (88.36%)	**3.54 (87.30%)**
2.00	2	0.06128	0.13985	1.07	2.07 (93.46%)	**2.06 (92.52%)**
			$\gamma_0 = 0.20$			
			***n* = 5**			
1.10	73	0.04080	0.41798	119.29	180.34 (51.18%)	**175.52 (47.14%)**
1.25	32	0.04488	0.40525	25.68	43.19 (68.19%)	**42.45 (65.30%)**
1.50	12	0.05057	0.38902	6.25	11.28 (80.48%)	**11.18 (78.88%)**
2.00	5	0.05647	0.37369	2.13	4.01 (88.26%)	**3.99 (87.32%)**
			***n* = 10**			
1.10	59	0.08355	0.34021	83.48	129.14 (54.70%)	**126.08 (51.03%)**
1.25	18	0.08976	0.32867	12.65	21.92 (73.28%)	**21.68 (71.38%)**
1.50	6	0.09636	0.31705	2.98	5.52 (85.23%)	**5.49 (84.23%)**
2.00	3	0.10096	0.30929	1.29	2.48 (92.25%)	**2.47 (91.47%)**

Table 3 *(Continued)*

$\gamma_0 = 0.05$						
τ	L	LCL	UCL	Zero-state ARL_1	Conditional Steady-state ARL'_1	**Cyclical Steady-state ARL'_1**
***n* = 15**						
1.10	49	0.10461	0.30989	62.78	98.81 (57.39%)	**96.73 (54.08%)**
1.25	13	0.11098	0.29973	7.97	14.15 (77.54%)	**14.02 (75.91%)**
1.50	5	0.11617	0.29178	2.02	3.82 (89.11%)	**3.80 (88.12%)**
2.00	2	0.12164	0.28368	1.10	2.12 (92.73%)	**2.11 (91.82%)**

From Table 3, the conditional and cyclical ARL_1 is larger than the zero-state ARL_1, especially for small values of τ. For instance, when $\gamma_0 = 0.05$, $n = 5$ and $\tau = 1.10$, under a zero-state assumption, practitioners would assume that the ARL_1 is 115.39. However, if these chart parameters are adopted for the steady-state condition, the ARL_1 increases to 175.10 and 170.37, respectively under conditional and cyclical assumptions. For ease of reference, the steady-state ARL_1s based on optimal chart parameters that minimize the zero-state ARL_1 is referred to as ARL'_1. There is a large difference between the zero-state ARL_1 and ARL'_1 when τ and n is small. As a result, under the steady-state condition, the zero-state ARL_1 gives an incorrect evaluation of the actual performance and will likely lead to a lack of confidence towards the chart.

There is a smaller difference between the zero-state ARL_1 and ARL'_1 for larger values of n and τ, although a higher percentage increase from the zero-state is shown. For example, when $\gamma_0 = 0.05$, $n = 5$ and $\tau = 1.10$, the difference between the zero-state ARL_1 with the conditional and cyclical ARL'_1 s are 59.71 and 54.98, respectively, but when $\gamma_0 = 0.05$, $n = 5$ and , the corresponding difference reduces to 1.76 and 1.74, respectively, while when $\gamma_0 = 0.05$, $n = 15$ and $\tau = 1.10$, the corresponding difference reduces to 33.73 and 31.88.

There are minimal differences between the conditional and cyclical ARL'_1, with the conditional ARL'_1 being slightly larger than the cyclical ARL'_1. For example, when $\gamma_0 = 0.05$, $n = 5$ and $\tau = 1.10$, the conditional and cyclical steady states ARL'_1 s are 175.10 and 170.37, respectively, with a difference of 2.70%. For larger values of τ, the differences are even smaller. For example, when $\gamma_0 = 0.05$, $n = 5$ and $\tau = 2.00$, the conditional and cyclical steady states ARL'_1 is 3.73 and 3.71, respectively, with a minimal difference of 0.54%.

Second Design. Alternative optimal chart parameters (L, LCL, UCL) are proposed to minimize the conditional and cyclical steady-state ARL_1s, unlike in first design where they are chosen to minimize the zero-state ARL_1. The chart parameters can be obtained through the following steps.

1. Specify γ_0, n and τ.
2. Set $L = 1$.
3. Numerically solve $ARL_0 = 370.4$ to obtain p. This is achieved by using numerical methods in the Scicoslab software to find the value of p to solve $\frac{1}{p\left(1-(1-p)^L\right)} = 370.4$.
4. By substituting p in Step 3 into Equation 3 and 4, the LCL and UCL are obtained.
5. Calculate the conditional (cyclical) ARL_1 from Equation 12 (Equation 13) with the current combination of (L, LCL, UCL).
6. Increase L by 1.
7. Repeat Steps 3 to 6 until the conditional (cyclical) ARL_1 for L+1 is larger than the conditional (cyclical) ARL_1 for L.

The (L, LCL, UCL) with the smallest conditional (cyclical) ARL_1 are considered the optimal chart parameters. In Step 3, p is obtained by solving the zero-state ARL_0, instead of solving the steady-state ARL_0 with $\delta = 0$ in Equation 12 or 13. This is to ensure that both the first and second designs have the same IC run length performance, so that a fair comparison can be made between these two designs.

The optimal chart parameters, conditional and cyclical ARL_1s when $\gamma_0 \in \{0.05, 0.10, 0.20\}$, $n \in \{5, 10, 15\}$ and $\tau \in \{1.10, 1.25, 1.50, 2.00\}$ are shown in Table 4. In parenthesis beside the ARL_1s are the percentage improvement compared to the first design in Table 3.

From Table 4, for smaller shift sizes, the conditional and cyclical ARL_1 adopting the optimal chart parameters that minimize the steady-state ARL_1 are smaller than the corresponding ARL'_1 that is based on the optimal parameters that minimize the zero-state ARL_1. For instance, when $\gamma_0 = 0.05$, $n = 5$ and $\tau = 1.10$, the conditional and cyclical ARL'_1 are 175.10 and 170.37, respectively, when the parameters from Table 3 are adopted. However, when the optimal parameters that minimizes the steady-state ARL_1 are adopted, the conditional and cyclical ARL_1 are 161.45 and 160.88, respectively. This shows a percentage improvement of 7.80% and 5.57%, respectively, when the correct optimal parameters are adopted. The reduction on ARL_1 shows that less average number of samples are required to detect the OOC condition. This results in quicker corrective action taken to repair the process and reduces the number of defective products produced due to an OOC process. For large shift sizes, there are minimal or no difference between the ARL_1 in Table 3 and 4.

Table 4

The optimal chart parameters and ARL_1 for the conditional and cyclical steady-state synthetic CV chart

	$\gamma_0 = 0.05$							
	Conditional Steady State				Cyclical Steady State			
				***n* = 5**				
τ	*L*	LCL	UCL	ARL_1	*L*	LCL	UCL	**ARL_1**
1.10	13	0.01264	0.09355	161.45 (7.80%)	14	0.01253	0.09382	**160.88 (5.57%)**
1.25	14	0.01253	0.09382	39.18 (3.19%)	15	0.01242	0.09407	**38.91 (2.26%)**
1.50	8	0.01343	0.09174	10.32 (1.43%)	8	0.01343	0.09174	**10.26 (1.06%)**
2.00	4	0.01467	0.08905	3.72 (0.27%)	4	0.01467	0.08905	**3.71 (0%)**
τ				***n* = 10**				
1.10	14	0.02305	0.07930	114.48 (6.47%)	15	0.02295	0.07846	**113.95 (4.69%)**
1.25	9	0.02371	0.07826	19.53 (2.50%)	9	0.02371	0.07826	**19.42 (2.02%)**
1.50	4	0.02501	0.07626	5.02 (0.59%)	5	0.02464	0.07682	**5.01 (0.40%)**
2.00	3	0.02550	0.07552	2.35 (0%)	3	0.02550	0.07552	**2.34 (0%)**
τ				***n* = 15**				
1.10	13	0.02804	0.07335	86.97 (5.75%)	15	0.02785	0.07361	**86.55 (4.22%)**
1.25	7	0.02886	0.07219	12.53 (-0.24%)	7	0.02886	0.07219	**12.47 (-0.65%)**
1.50	3	0.03008	0.07052	3.50 (0%)	3	0.03008	0.07052	**3.40 (2.58%)**
2.00	2	0.03070	0.06968	2.06 (0%)	2	0.03070	0.06968	**2.05 (0%)**
				$\gamma_0 = 0.10$				
τ				***n* = 5**				
1.10	13	0.02524	0.18865	162.36 (7.85%)	14	0.02501	0.18921	**161.78 (5.63%)**
1.25	14	0.02501	0.18921	39.66 (3.53%)	15	0.02480	0.18973	**39.38 (2.45%)**
1.50	8	0.02681	0.18493	10.48 (1.32%)	8	0.02681	0.18493	**10.42 (1.04%)**
2.00	4	0.02928	0.17940	3.77 (0.26%)	4	0.02928	0.17940	**3.76 (0%)**

Table 4 *(Continued)*

	$\gamma_0 = 0.05$							
	Conditional Steady State				Cyclical Steady State			
	L	LCL	UCL	ARL_1	*L*	LCL	UCL	$\mathbf{ARL_1}$
τ				***n* = 10**				
1.10	14	0.04600	0.15947	115.66 (7.26%)	15	0.04580	0.15980	**115.12 (4.92%)**
1.25	9	0.04732	0.15734	19.88 (2.41%)	10	0.04700	0.15785	**19.78 (1.84%)**
1.50	4	0.04992	0.15326	5.12 (0.58%)	5	0.04918	0.15440	**5.09 (0.59%)**
2.00	3	0.05090	0.15175	2.37 (0%)	3	0.05090	0.15175	**2.37 (0%)**
				***n* = 15**				
1.10	13	0.05596	0.14732	88.12 (5.68%)	15	0.05559	0.14785	**87.68 (4.25%)**
1.25	7	0.05761	0.14496	12.78 (1.99%)	7	0.05761	0.14496	**12.72 (1.70%)**
1.50	3	0.06004	0.14155	3.55 (0.28%)	3	0.06004	0.14155	**3.54 (0%)**
2.00	2	0.06128	0.13985	2.07 (0%)	2	0.06128	0.13985	**2.06 (0%)**
				$\gamma_0 = 0.20$				
τ				***n* = 5**				
1.10	13	0.05007	0.39038	166.11 (7.89%)	13	0.05007	0.39038	**165.54 (5.69%)**
1.25	14	0.04961	0.39164	41.67 (3.65%)	15	0.04919	0.39280	**41.38 (2.52%)**
1.50	8	0.05320	0.38201	11.15 (1.15%)	8	0.05320	0.38201	**11.08 (0.89%)**
2.00	4	0.05810	0.36966	4.01 (0%)	5	0.05647	0.37369	**3.99 (0%)**
τ				***n* = 10**				
1.10	13	0.09163	0.32531	120.45 (6.73%)	15	0.09080	0.32680	**119.90 (4.90%)**
1.25	9	0.09383	0.32143	21.36 (2.55%)	10	0.09319	0.32255	**21.24 (2.03%)**
1.50	5	0.09754	0.31504	5.49 (0.54%)	5	0.09754	0.31504	**5.47 (0.36%)**
2.00	3	0.10097	0.30929	2.48 (0%)	3	0.10096	0.30929	**2.47 (0%)**

The difference between the conditional and cyclical ARL_1 by adopting the optimal chart parameters that minimize the steady-state ARL_1 are smaller compared to the one in Table 3. For example, for $\gamma_0 = 0.05$, $n = 5$ and $\tau = 1.10$, the conditional and cyclical ARL_1 are 161.45 and 160.88 in Table 4 (a difference of 0.35%), but for Table 3, they are 175.10 and 170.37 (a difference of 2.70%). Similar with Knoth (2016), the optimal *L* in Table 4 is smaller than the optimal *L* in Table 3, especially for small shift sizes. For example, when $\gamma_0 = 0.05$, $n = 5$ and $\tau = 1.10$, the optimal *L* in Table 4 is 13 and 14 under the conditional and cyclical assumptions, respectively, while the corresponding optimal *L* in Table 3 is 73. Meanwhile, a larger LCL and smaller UCL is observed in Table 4, compared with Table 3. For example, when $\gamma_0 = 0.05$, $n = 5$ and $\tau = 1.10$, the optimal (LCL, UCL) in Table 4 is (0.01264, 0.09355) and (0.01253, 0.09832) under the conditional and cyclical assumptions, respectively, while the corresponding optimal (LCL, UCL) in Table 3 is (0.01031, 0.09943). Hence, the range of CV values for a sample to be identified as conforming has been reduced and will likely result in an increase in the occurrence of non-conforming samples. From Equation 5, a larger LCL and smaller UCL will decrease the probability for *A*, thus increasing the probability of non-conforming samples. For example, when $\gamma_0 = 0.05$, $n = 5$ and $\tau = 1.10$, the probability of a non-conforming sample is 0.02659 and 0.01784 for the conditional and cyclical assumptions in Table 4, respectively, while the corresponding probability of a non-conforming sample in Table 3 is 0.01367.

Although the second design results in better steady-state performance than the first design for small shift sizes, the ARL_1 is still quite large compared with the zero-state ARL_1. As expected, the zero-state ARL_1 does not give a true picture on the chart's performance under the steady-state condition.

Performance Comparisons

The synthetic CV chart is compared with the run rules, EWMA and Shewhart CV charts in this section. Castagliola et al. (2013) and Castagliola et al. (2011) gave a thorough discussion on the run rules and EWMA CV charts, respectively.

The zero-state ARL_1 of these charts, for $\gamma_0 \in \{0.05, 0.10, 0.20\}$, $n \in \{5,10,15\}$ and $\tau \in \{1.1,1.2,1.5,2.0\}$ are shown in Table 5. In parenthesis are the percentage of improvement of the run rules, EWMA and Shewhart CV charts compared to the synthetic CV chart, where a negative percentage shows that the synthetic CV chart performs better. The optimal *L* in Table 3 is adopted for the synthetic CV chart. For instance, when *n* = 5 and $\gamma_0 = 0.05$, L = 73, 30, 12 and 5 for $\tau = 1.10, 1.25, 1.50$ and 2.00, respectively. Besides that, it is assumed that the initial state for the synthetic CV chart is state 0 since that is the assumption for the zero-state design of the chart.

From Table 5, the zero-state synthetic CV chart outperforms the Shewhart CV chart for all γ_0, *n* and τ. This is especially so for small *n* and τ, where the difference in ARL_1

is quite large. The zero-state synthetic CV chart outperforms the zero-state run rules CV chart for most cases, except when $\tau = 1.10$ for n = 5, where the run rules chart slightly outperforms the synthetic chart. The zero-state EWMA chart outperforms the zero-state synthetic chart, except when shift sizes are large.

Although the synthetic CV chart seems to outperform the run rules CV chart in most cases, the comparison is based on a zero-state assumption. Note that the zero-state synthetic CV chart assumes that the initial state is zero. Although a zero initial state results in the best performance, the probability for a zero initial state is very small in a steady-state scenario. For example, from Table 2, when $\gamma_0 = 0.05$, $n = 5$ and $\tau = 1.10$, the conditional and cyclical probability of a zero initial state are 0.00482 and 0.00685, respectively. This shows that there is a less than 1% chance for the occurrence of the zero-initial state. Hence, in Table 6, the conditional and cyclical steady-state ARL_1s for synthetic, run rules, EWMA and Shewhart CV charts are compared. In parenthesis are the percentage of improvement of the run rules, EWMA and Shewhart CV charts compared to the synthetic CV chart, where a negative percentage shows that the synthetic CV chart performs better.

By comparing Tables 5 and 6 for the run rules and EWMA charts, negligible differences are shown between the zero-state and steady-state ARL_1s, since these two charts does not involve any head-start feature, while the zero-state and steady-state Shewhart CV charts are the same, as no assumptions are required for the initial state. Note that in Table 6, the same L as that in Table 4 is adopted for the synthetic chart. For example, for n = 5 and $\gamma_0 = 0.05$, L = 13, 14, 8, and 4 for $\tau = 1.10, 1.25, 1.50$ and 2.00, respectively for the conditional steady-state, while L = 14, 15, 8 and 4 for $\tau = 1.10, 1.25, 1.50$ and 2.00, respectively during cyclical steady-states.

From Table 6, the run rules CV chart outperforms the synthetic CV chart for all n, γ_0 and τ, especially for small n and τ. However, for large n and τ, the synthetic and run rules CV charts shows comparable steady-state performance, with the run rules CV chart showing slightly better performance. Hence, the synthetic CV chart outperforms the run rules CV chart only for a zero-state assumption and not under the steady-state scenario. In fact, for numerous cases, the synthetic CV chart is not even better than the basic Shewhart CV chart. Since it is more difficult to implement the synthetic CV chart, practitioners would rather implement the Shewhart CV chart. Furthermore, the steady-state EWMA CV chart outperforms the steady-state synthetic CV chart for all n, γ_0 and τ. Hence, the synthetic CV chart does not show good performance under the steady-state scenario.

Table 5

A comparison of the zero-state ARL_1 of the synthetic, run rules and Shewhart CV charts for $\gamma_0 \in \{0.05, 0.10, 0.20\}$, $n \in \{5,10,15\}$ *and* $\tau \in \{1.1, 1.2, 1.5, 2.0\}$

n = 5												
τ	$\gamma_0 = 0.05$				$\gamma_0 = 0.10$				$\gamma_0 = 0.20$			
	Synthetic	Run Rules	EWMA	Shewhart	Synthetic	Run Rules	EWMA	Shewhart	Synthetic	Run Rules	EWMA	**Shewhart**
1.10	115.39	113.77	55.05	159.86	116.16	113.88	55.21	160.64	119.29	111.57	55.85	**163.95**
		(1.40%)	(52.29%)	(-38.54%)		(1.96%)	(52.47%)	(-38.29%)		(6.47%)	(53.18%)	**(-37.44%)**
1.25	24.02	30.65	15.30	43.55	24.34	30.66	15.37	44.08	25.68	30.82	15.67	**44.98**
		(-27.60%)	(36.30%)	(-81.31%)		(-25.97%)	(36.85%)	(-81.10%)		(-20.02%)	(38.98%)	**(-75.16%)**
1.50	5.76	9.01	5.65	10.57	5.85	9.01	5.70	10.76	6.25	9.36	5.91	**11.09**
		(-56.42%)	(1.91%)	(-83.51%)		(-54.02%)	(2.56%)	(-83.93%)		(-49.76%)	(5.44%)	**(-77.44%)**
2.00	1.97	3.53	2.34	2.89	2.00	3.53	2.37	2.95	2.13	3.72	2.51	**3.05**
		(-79.19%)	(-18.78%)	(-46.70%)		(-76.50%)	(-18.50%)	(-47.50%)		(-74.65%)	(-17.84%)	**(-43.19%)**
n = 10												
τ	$\gamma_0 = 0.05$				$\gamma_0 = 0.10$				$\gamma_0 = 0.20$			
	Synthetic	Run Rules	EWMA	Shewhart	Synthetic	Run Rules	EWMA	Shewhart	Synthetic	Run Rules	EWMA	**Shewhart**
1.10	78.87	90.88	32.08	120.27	79.77	90.60	32.30	121.32	83.48	89.68	33.17	**125.69**
		(-15.23%)	(59.33%)	(-52.49%)		(-13.58%)	(59.51%)	(-52.09%)		(-7.43%)	(60.27%)	**(-50.56%)**
1.25	11.48	17.44	8.50	22.94	11.71	17.56	8.57	23.37	12.65	18.07	8.88	**24.11**
		(-51.92%)	(25.96%)	(-99.83%)		(-49.96%)	(26.81%)	(-99.57%)		(-42.85%)	(29.80%)	**(-90.59%)**
1.50	2.71	4.75	3.14	4.78	2.76	4.81	3.18	4.89	2.98	5.05	3.35	**5.09**
		(-75.28%)	(-15.87%)	(-76.38%)		(-74.28%)	(-15.22%)	(-77.17%)		(-69.46%)	(-12.42%)	**(-70.81%)**
2.00	1.22	2.36	1.41	1.52	1.24	2.38	1.43	1.55	1.29	2.46	1.51	**1.60**
		(-93.44%)	(-15.57%)	(-24.59%)		(-91.94%)	(-15.32%)	(-25.00%)		(-90.70%)	(-17.05%)	**(-24.03%)**
τ	$\gamma_0 = 0.05$				$\gamma_0 = 0.10$				$\gamma_0 = 0.20$			
	Synthetic	Run Rules	EWMA	Shewhart	Synthetic	Run Rules	EWMA	Shewhart	Synthetic	Run Rules	EWMA	**Shewhart**
n = 15												
τ	$\gamma_0 = 0.05$				$\gamma_0 = 0.10$				$\gamma_0 = 0.20$			
	Synthetic	Run Rules	EWMA	Shewhart	Synthetic	Run Rules	EWMA	Shewhart	Synthetic	Run Rules	EWMA	**Shewhart**
1.10	58.48	73.53	23.58	95.85	59.32	73.53	23.78	96.92	62.78	73.56	24.55	**101.34**
		(-25.74%)	(59.68%)	(-63.90%)		(-23.95%)	(59.91%)	(-63.39%)		(-17.17%)	(60.90%)	**(-61.42%)**
1.25	7.18	11.65	6.14	14.83	7.33	11.79	6.21	15.16	7.97	12.31	6.46	**15.71**
		(-62.26%)	(14.48%)	(-106.55%)		(-60.85%)	(15.28%)	(-106.82%)		(-54.45%)	(18.95%)	**(-97.11%)**
1.50	1.86	3.40	2.29	3.02	1.89	3.45	2.33	3.09	2.02	3.62	2.46	**3.22**
		(-82.80%)	(-23.12%)	(-62.37%)		(-82.54%)	(-23.28%)	(-63.49%)		(-79.21%)	(-21.78%)	**(-59.41%)**
2.00	1.07	2.11	1.15	1.19	1.07	2.12	1.16	1.21	1.10	2.16	1.21	**1.23**
		(-97.20%)	(-7.48%)	(-11.21%)		(-98.13%)	(-8.41%)	(-13.08%)		(-96.36%)	(-10.00%)	**(-11.82%)**

Table 6

The conditional and cyclical steady-state ARL_1 of the synthetic, run rules, EWMA and Shewhart CV charts for $\tau \in \{1.1, 1.2, 1.5, 2.0\}$

τ	$n = 5$														
	$\gamma_0 = 0.05$								$\gamma_0 = 0.10$						
	Synthetic		Run Rules		EWMA		Shewhart		Synthetic		Run Rules		EWMA		Shewhart
	Cond	Cyc	Cond	Cyc	Cond	Cyc			Cond	Cyc	Cond	Cyc	Cond	Cyc	
1.10	161.45	160.88	113.02 (30.00%)	113.03 (29.74%)	52.70 (67.36%)	52.77 (67.20%)	159.86 (0.98%)		162.36	161.78	112.52 (30.70%)	112.52 (30.45%)	52.89 (67.42%)	52.95 (67.27%)	160.64 (1.06%)
1.25	39.18	38.91	30.28 (22.72%)	30.28 (22.18%)	14.82 (62.17%)	14.83 (61.89%)	43.55 (-11.15%)		39.66	39.38	30.30 (23.60%)	30.30 (23.06%)	14.90 (62.43%)	14.90 (62.16%)	44.08 (-11.14%)
1.50	10.32	10.26	8.83 (14.44%)	8.83 (13.94%)	5.52 (46.51%)	5.52 (46.20%)	10.57 (-2.42%)		10.48	10.42	8.90 (15.08%)	8.90 (14.59%)	5.57 (46.85%)	5.57 (46.55%)	10.76 (-2.67%)
2.00	3.72	3.71	3.43 (7.80%)	3.43 (7.55%)	2.32 (37.63%)	2.32 (37.47%)	2.89 (22.31%)		3.77	3.76	3.47 (7.96%)	3.47 (7.71%)	2.35 (37.67%)	2.35 (37.50%)	2.95 (21.75%)
	$n = 10$														
τ	$\gamma_0 = 0.05$								$\gamma_0 = 0.10$						
	Synthetic		Run Rules		EWMA		Shewhart		Synthetic		Run Rules		EWMA		Shewhart
	Cond	Cyc	Cond	Cyc	Cond	Cyc			Cond	Cyc	Cond	Cyc	Cond	Cyc	
1.10	114.48	113.95	90.21 (21.20%)	90.22 (20.82%)	30.73 (73.16%)	30.80 (72.97%)	120.27 (-5.06%)		115.66	115.12	89.94 (22.24%)	89.94 (21.87%)	30.99 (73.21%)	31.02 (73.05%)	121.32 (-4.89%)
1.25	19.53	19.42	17.18 (12.03%)	17.18 (11.53%)	8.23 (57.86%)	8.16 (57.98%)	22.94 (-17.46%)		19.88	19.78	17.30 (12.98%)	17.30 (12.54%)	8.31 (58.20%)	8.24 (58.34%)	23.37 (-17.56%)
1.50	5.02	5.01	4.63 (7.77%)	4.63 (7.58%)	3.09 (38.45%)	3.09 (38.32%)	4.78 (4.78%)		5.12	5.09	4.69 (8.40%)	4.69 (7.86%)	3.13 (38.87%)	3.13 (38.51%)	4.89 (4.49%)
2.00	2.35	2.34	2.29 (2.55%)	2.29 (2.14%)	1.40 (40.43%)	1.41 (39.74%)	1.52 (35.32%)		2.37	2.37	2.30 (2.95%)	2.30 (2.95%)	1.42 (40.08%)	1.43 (39.66%)	1.55 (34.60%)
	$n = 15$														
τ	$\gamma_0 = 0.05$								$\gamma_0 = 0.10$						
	Synthetic		Run Rules		EWMA		Shewhart		Synthetic		Run Rules		EWMA		Shewhart
	Cond	Cyc	Cond	Cyc	Cond	Cyc			Cond	Cyc	Cond	Cyc	Cond	Cyc	
1.10	86.97	86.55	72.94 (16.13%)	72.94 (15.73%)	22.65 (73.96%)	22.27 (74.27%)	95.85 (-10.21%)		88.12	87.68	72.94 (17.23%)	72.94 (16.81%)	22.84 (74.08%)	22.86 (73.93%)	96.92 (-9.99%)
1.25	12.53	12.47	11.45 (8.62%)	11.45 (8.18%)	5.96 (52.43%)	5.91 (52.61%)	14.83 (-18.36%)		12.78	12.72	11.58 (9.39%)	11.58 (8.96%)	6.01 (52.97%)	5.97 (53.07%)	15.16 (-18.62%)
1.50	3.50	3.40	3.31 (5.43%)	3.31 (2.65%)	2.26 (35.43%)	2.26 (33.53%)	3.02 (13.71%)		3.55	3.54	3.35 (5.63%)	3.35 (5.37%)	2.30 (35.21%)	2.30 (35.03%)	3.09 (12.96%)
2.00	2.06	2.05	2.04 (0.97%)	2.04 (0.49%)	1.15 (44.17%)	1.15 (43.90%)	1.19 (42.23%)		2.07	2.06	2.05 (0.97%)	2.05 (0.49%)	1.16 (43.96%)	1.16 (43.69%)	1.21 (41.55%)

Table 6 *(Continued)*

τ	$\gamma_0 = 0.20$						
	Synthetic		Run Rules		EWMA		Shewhart
	Cond	Cyc	Cond	Cyc	Cond	Cyc	
1.10	166.11	165.54	110.83 (33.28%)	110.83 (33.05%)	53.60 (67.73%)	52.50 (68.29%)	163.95 (1.30%)
1.25	41.67	41.38	30.45 (26.93%)	30.45 (26.41%)	15.20 (63.52%)	15.12 (63.46%)	44.98 (-7.94%)
1.50	11.15	11.08	9.18 (17.67%)	9.18 (17.15%)	5.77 (48.25%)	5.79 (47.74%)	11.09 (0.54%)
2.00	4.01	3.99	3.61 (9.98%)	3.61 (9.52%)	2.49 (37.91%)	2.49 (37.59%)	3.05 (23.94%)
			***n* = 10**				
τ	$\gamma_0 = 0.20$						
	Synthetic		Run Rules		EWMA		Shewhart
	Cond	Cyc	Cond	Cyc	Cond	Cyc	
1.10	120.45	119.90	89.02 (26.09%)	89.03 (25.75%)	31.86 (73.55%)	31.89 (73.40%)	125.69 (-4.35%)
1.25	21.36	21.24	17.80 (16.67%)	17.80 (16.20%)	8.61 (59.69%)	8.56 (59.70%)	24.11 (-12.87%)
1.50	5.49	5.47	4.93 (10.20%)	4.93 (9.87%)	3.29 (40.07%)	3.29 (39.85%)	5.09 (7.29%)
2.00	2.48	2.47	2.38 (4.03%)	2.38 (3.64%)	1.51 (39.11%)	1.51 (38.87%)	1.60 (35.48%)
			***n* = 15**				
τ	$\gamma_0 = 0.20$						
	Synthetic		Run Rules		EWMA		Shewhart
	Cond	Cyc	Cond	Cyc	Cond	Cyc	
1.10	92.74	92.30	72.97 (21.32%)	72.97 (20.94%)	23.60 (74.55%)	23.62 (74.41%)	101.34 (-9.27%)
1.25	13.81	13.75	12.10 (12.38%)	12.10 (12.00%)	6.27 (54.60%)	6.23 (54.69%)	15.71 (-13.76%)
1.50	3.79	3.78	3.52 (7.12%)	3.52 (6.88%)	2.42 (36.15%)	2.42 (35.98%)	3.22 (15.04%)
2.00	2.12	2.11	2.08 (1.89%)	2.08 (1.42%)	1.21 (42.92%)	1.21 (42.65%)	1.23 (41.98%)

CONCLUSIONS

This paper studies the performance of the synthetic CV chart under the more realistic steady-state condition, where the assignable cause which results in an OOC condition happens after the process has been operating for some time. Under the steady-state condition, the advantage of the head-start feature has faded away. This paper contributes to the literature by highlighting large differences between the steady-state and zero-state performances, especially for small shift sizes and sample sizes. Hence, the zero-state performance is not an accurate representation for the synthetic CV chart's performance. Practitioners should be cautious in evaluating the performance of the synthetic CV chart based on its zero-state performance.

This paper also proposes an alternative design for the synthetic CV chart based on its steady-state performance. The alternative design results in an improvement in the steady-state performance of the synthetic CV chart. The proposed design is useful for practitioners who intend to adopt the synthetic CV chart to monitor a steady-state process, which is more realistic in most practical applications since processes are usually stable in the beginning. The synthetic CV chart does not show better steady-state performance compared to the run rules and Shewhart CV charts for small sample sizes and shift sizes, while comparable performance is shown for large sample sizes and shift sizes. Among the charts under comparison, the EWMA CV chart has the best steady-state performance.

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REFERENCES

Calzada, M. E., & Scariano, S. M. (2013). A synthetic control chart for the coefficient of variation. *Journal of Statistical Computation and Simulation*, *83*(5), 853-867. https://doi.org/10.1080/00949655.2011.639772

Castagliola, P., Achouri, A., Taleb, H., Celano, G., & Psarakis, S. (2013). Monitoring the coefficient of variation using control charts with run rules. *Quality Technology and Quantitative Management*, *10*(1), 75-94. https://doi.org/10.1080/16843703.2013.11673309

Castagliola, P., Celano, G., & Psarakis, S. (2011). Monitoring the coefficient of variation using EWMA charts. *Journal of Quality Technology*, *43*(3), 249-265. https://doi.org/10.1080/00224065.2011.11917861

Curto, J. D., & Pinto, J. C. (2009). The coefficient of variation asymptotic distribution in the case of non-iid random variables. *Journal of Applied Statistics*, *36*(1), 21-32. https://doi.org/10.1080/02664760802382491

Davis, R. B., & Woodall, W. H. (2002). Evaluating and improving the synthetic control chart. *Journal of Quality Technology*, *34*(2), 200-208. https://doi.org/10.1080/00224065.2002.11980146

Kang, C. W., Lee, M. S., Seong, Y. J., & Hawkins, D. M. (2007). A control chart for the coefficient of variation. *Journal of Quality Technology*, *39*(2), 151-158. https://doi.org/10.1080/00224065.2007.11917682

Khan, Z., Razali, R. B., Daud, H., Nor, N. M., & Fotuhi-Firuzabad, M. (2017). The control chart technique for the detection of the problem of bad data in state estimation power system. *Pertanika Journal of Science & Technology*, *25*(3), 825-834.

Kinat, S., Amin, M., & Mahmood, T. (2020). GLM-based control charts for the inverse Gaussian distributed response variable. *Quality and Reliability Engineering International*, *36*(2), 765-783. https://doi.org/10.1002/qre.2603

Knoth, S. (2016). The case against the use of the synthetic control charts. *Journal of Quality Technology*, *48*(2), 178-195. https://doi.org/10.1080/00224065.2016.11918158

Machado, M. A. G., & Costa, A. F. B. (2014). Some comments regarding the synthetic chart. *Communications in Statistics - Theory and Methods*, *43*(14), 2897-2906. https://doi.org/10.1080/03610926.2012.683128

Mahmood, T., & Abbasi, S. A. (2021). Efficient monitoring of coefficient of variation with an application to chemical reactor process. *Quality and Reliability Engineering International*, *37*(3), 1135-1149. https://doi.org/10.1002/qre.2785

Marchant, C., Leiva, V., Cysneiros, F. J. A., & Liu, S. (2018). Robust multivariate control charts based on Birnbaum-Saunders distributions. *Journal of Statistical Computation and Simulation*, *88*(1), 182-202. https://doi.org/10.1080/00949655.2017.1381699

Mim, F. N., Saha, S., Khoo, M. B. C., & Khatun, M. (2019). A side-sensitive modified group runs control chart with auxiliary information to detect process mean shifts. *Pertanika Journal of Science & Technology*, *27*(2), 847-866.

Montgomery, D. C. (2019). *Introduction to statistical quality control*. John Wiley and Sons Inc.

Pang, W. K., Yu, B. W., Troutt, M. D., & Shui, H. H. (2008). A simulation-based approach to the study of coefficient of variation of dividend yields. *European Journal of Operational Research*, *189*(2), 559-569. https://doi.org/10.1016/j.ejor.2007.05.032

Rakitzis, A. C., Chakraborti, S., Shongwe, S. C., Graham, M. A., & Khoo, M. B. C. (2019). An overview of synthetic-type control charts: Techniques and methodology. *Quality and Reliability Engineering International*, *35*(7), 2081-2096. https://doi.org/10.1002/qre.2491

Shongwe, S. C., & Graham, M. A. (2017). A letter to the editor about "Machado, MAG and Costa AFB (2014), some comments regarding the synthetic chart. Communications in Statistics---Theory and Methods, 43 (14), 2897–2906". *Communications in Statistics - Theory and Methods, 46*(21), 10476-10480. https://doi.org/10.1080/03610926.2016.1236958

Shongwe, S. C., & Graham, M. A. (2019). Some theoretical comments regarding the run-length properties of the synthetic and runs-rules X¯ monitoring schemes–part 2: Steady-state. *Quality Technology & Quantitative Management, 16*(2), 190-199. https://doi.org/10.1080/16843703.2017.1389141

Teoh, W. L., Khoo, M. B. C., Yeong, W. C., & Teh, S. Y. (2016). A comparison between the performances of synthetic and EWMA charts for monitoring the coefficient of variation. *Journal of Scientific Research and Development*, *3*(1), 16-20.

Wu, Z., & Spedding, T. A. (2000). A synthetic control chart for detecting small shifts in the process mean. *Journal of Quality Technology*, *32*(1), 32-38. https://doi.org/10.1080/00224065.2000.11979969

Wu, Z., Ou, Y., Castagliola, P., & Khoo, M. B. C. (2010). A combined synthetic & X chart for monitoring the process mean. *International Journal of Production Research*, *48*(24), 7423-7436. https://doi.org/10.1080/00207540903496681

Yeong, W. C., Khoo, M. B. C., Tham, L. K., Teoh, W. L., & Rahim, M. A. (2017). Monitoring the coefficient of variation using a variable sampling interval EWMA chart. *Journal of Quality Technology*, *49*(4), 380-401. https://doi.org/10.1080/00224065.2017.11918004

Yeong, W. C., Lim, S. L., Khoo, M. B. C., Chuah, M. E., & Lim, A. J. X. (2018). The economic and economic-statistical designs of the synthetic chart for the coefficient of variation. *Journal of Testing and Evaluation*, *46*(3), 1175-1195. https://doi.org/10.1520/JTE20160500

You, H. W. (2018). Design of the side sensitive group runs chart with estimated parameters based on expected average run length. *Pertanika Journal of Science & Technology*, *26*(3), 847-866.

Zhang, J., Li, Z., & Wang, Z. (2018). Control chart for monitoring the coefficient of variation with an exponentially weighted moving average procedure. *Quality and Reliability Engineering International*, *34*(2), 188-202. https://doi.org/10.1002/qre.2247